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THERMAL REGIME OF MOIST CONCRETE WALLS SUBMERGED INTO THE GROUND OF STRUCTURES UNDER CONDITIONS OF CONVECTIVE DRYING

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The article provides the solution of the heat conduction for a semibounded massif with the boundary condition of the third kind taking into account the effect of evaporation on the heat-exchange surface.

Drying is a complex process of non-steady-state heat and moisture exchange which, according to the analytical theory [1, 2], is described by the system of differential equations

$$\frac{\partial t}{\partial \tau} = a\nabla^2 t + \zeta \frac{r}{c} \frac{\partial u}{\partial \tau},$$
$$\frac{\partial u}{\partial \tau} = a_m \nabla^2 u + a_m \delta \nabla^2 t. \qquad (1)$$

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At present we lack strict correlations for δ and ζ in the case of many materials. This limits the application of the solutions of system (1). In engineering practice, the separate method of heat-engineering calculation of moist bodies is therefore often used. Its essence [1, 3] consists in determining the functional dependence for the flow density of the moisture, and subsequently solving the equation of heat conductivity with boundary conditions taking the discharge of heat for evaporation into account.

For determining the flow density of the moisture, we analyze the equation of moisture conductivity for the case of mass transfer in a wall submerged into the ground of a structure. In drying, the moisture flow in the wall has to be directed toward the premises. The heat flow must not change this direction. The thermogradient coefficient is very small, and therefore the equation of moisture conductivity can be represented in the first approximation in the form

$$\frac{\partial u(x,\ \tau)}{\partial \tau} = a_m \frac{\partial^2 u(x,\ \tau)}{\partial x^2} \quad (\tau > 0,\ 0 \leqslant x \leqslant b). \tag{2}$$

We determine the moisture content of the surface layer of the wall (Fig. 1a). For that, we must solve Eq. (2) with the boundary conditions

$$\frac{\partial u(0,\ \tau)}{\partial x} = 0, \tag{3}$$

$$\frac{\partial u(b,\ \tau)}{\partial x} = -\frac{\beta}{a_m}[u(b,\ \tau) - u_e], \tag{4}$$

$$u(x,\ 0) = u_0. \tag{5}$$

For further analysis we use the solution of the heat-conduction equation with similar boundary conditions as given in [3]. We note that for the process of organized convective drying of concrete walls, $Fo_m < 0.1$ is characteristic. In that case we may write

$$\frac{u_0 - u(b,\ \tau)}{u_0 - u_e} = 1 - \exp(Bi_m^2 Fo_m)\operatorname{erfc}(Bi_m \sqrt{Fo_m}). \tag{6}$$

If we represent the density of the moisture flow on the wall surface in the form

$$j = \beta\rho[u(b,\ \tau) - u_e] \tag{7}$$

and solve (6) and (7) jointly, we obtain

$$j = \beta\rho(u_0 - u_e)\exp(Bi_m^2 Fo_m)\operatorname{erfc}(Bi_m \sqrt{Fo_m}) \tag{8}$$

or

$$j = m_0 \exp(Bi_m^2 Fo_m)\operatorname{erfc}(Bi_m \sqrt{Fo_m}). \tag{9}$$

For compiling the theoretical diagram of the problem of heating moist walls submerged into the ground of structures, we examine the most characteristic case of their design when the load-bearing part (the concrete lining) has no heat insulation. In view of the fact that the heat-engineering characteristics (thermal conductivity, specific heat, and thermal diffusivity) of concretes and grounds of medium moisture content are equivalent, we will consider the concrete and the ground massif around the structure as a homogeneous and isotropic body.

In that case, the problem is written in the following way. Given is a semibounded body (Fig. 1b) with the initial temperature t_0. The bounding surface is washed by air with temperature t_a. The heat exchange characterized by the coefficient α occurs according to Newton's law. The moisture exchange on the surface is characterized by variable density j. We have to find the temperature distribution in the body at an arbitrary instant of time.

Finding the temperature field in the wall is bound up with the solution of the differential heat-conduction equation

$$\frac{\partial t(x,\ \tau)}{\partial \tau} = a \frac{\partial^2 t(x,\ \tau)}{\partial x^2} \quad (\tau > 0,\ 0 < x < \infty) \tag{10}$$

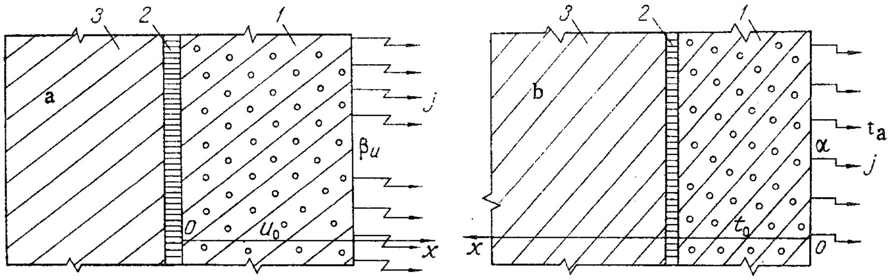

Fig. 1. Theoretical diagram for the problem of drying (a) and heating (b) of a moist wall submerged into the ground of a structure: 1) concrete wall; 2) waterproofing insulation; 3) ground massif.

with the boundary conditions

$$t(x,\ 0)=t_0, \tag{11}$$

$$\lambda\frac{\partial t(0,\ \tau)}{\partial x}+\alpha\,[t_a-t(0,\ \tau)]-jr=0, \tag{12}$$

$$\frac{\partial t(\infty,\ \tau)}{\partial x}=0. \tag{13}$$

Taking Eq. (9) into account, we write Eq. (12) as follows:

$$\lambda\frac{\partial t(0,\ \tau)}{\partial x}+\alpha\,[t_a-t(0,\ \tau)]-m_0 r\exp(k^2\tau)\operatorname{erfc}(k\sqrt{\tau}). \tag{14}$$

If we apply the Laplace transform to Eq. (10), we obtain its solution in the images with the initial condition (11) in the form

$$t_L(x,\ p)=\frac{t_0}{p}+Ae^{\sqrt{p/a}x}+Be^{-\sqrt{p/a}x}. \tag{15}$$

The boundary conditions (13) and (14) in the images have the form

$$\lambda t_L'(0,\ p)+\alpha\frac{t_a}{p}-\alpha t_L(0,\ p)-\frac{m_0 r}{\sqrt{p}(\sqrt{p}+k)}=0, \tag{16}$$

$$t_L'(\infty,\ p)=0. \tag{17}$$

It follows from condition (17) that A = 0. From the requirement that the solution of (15) satisfy the boundary condition (16), we determine the coefficient B:

$$B=\frac{\alpha(t_a-t_0)}{p(\alpha+\lambda\sqrt{p/a})}-\frac{m_0 r}{(\alpha+\lambda\sqrt{p/a})\sqrt{p}(\sqrt{p}+k)}. \tag{18}$$

Thus, the solution of Eq. (10) in images is written in the form

$$t_L(x,\ p)=\frac{t_0}{p}+\frac{\alpha(t_a-t_0)}{p(\alpha+\lambda\sqrt{p/a})}e^{-\sqrt{p/a}x}-\frac{m_0 r}{(\alpha+\lambda\sqrt{p/a})\sqrt{p}(\sqrt{p}+k)}e^{-\sqrt{p/a}x}. \tag{19}$$

In accordance with the table of images [3], the preimage of the second term in the right-hand part of (19) has the form

$$L^{-1}\left[\frac{\alpha(t_a-t_0)}{p(\alpha+\lambda\sqrt{p/a})}e^{-\sqrt{p/a}x}\right]=(t_a-t_0)\left[\operatorname{erfc}\frac{x}{2\sqrt{a\tau}}-\exp(Hx+H^2a\tau)\operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}}+H\sqrt{a\tau}\right)\right]. \tag{20}$$

To find the preimage of the third term in the right-hand part of (19), we transform the latter under the condition that $(\alpha/\lambda)\sqrt{a}\neq k$.

An analysis of numerous regimes of convective drying of massive concrete walls shows that they are characterized by $k = (0.001\text{-}0.003) 1/\sqrt{c}$ and $(\alpha/\lambda)\sqrt{a} = (0.005\text{-}0.02)\ 1/\sqrt{c}$.

Thus

$$\frac{m_0 r}{(\alpha+\lambda\sqrt{p/a})\sqrt{p}(\sqrt{p}+k)} e^{-\sqrt{p/a}x} = m_0 r \frac{\frac{1}{\left(\alpha\frac{\sqrt{a}}{\lambda}-k\right)}\left(\frac{\alpha\sqrt{a}}{\lambda}+\sqrt{p}-\sqrt{p}-k\right)}{(\alpha+\lambda\sqrt{p/a})\sqrt{p}(\sqrt{p}+k)} e^{-\sqrt{p/a}x} =$$

$$= \frac{m_0 r}{\left(\alpha\frac{\sqrt{a}}{\lambda}-k\right)}\left[\frac{\frac{\sqrt{a}}{\lambda}\left(\alpha+\lambda\sqrt{p/a}\right)}{(\alpha+\lambda\sqrt{p/a})\sqrt{p}(\sqrt{p}+k)} - \frac{(\sqrt{p}+k)}{(\alpha+\lambda\sqrt{p/a})\sqrt{p}(\sqrt{p}+k)}\right] e^{-\sqrt{p/a}x} = \frac{m_0 r}{\left(\alpha\frac{\sqrt{a}}{\lambda}-k\right)}$$

$$\times\left[\frac{\sqrt{a}/\lambda}{\sqrt{p}(\sqrt{p}+k)} - \frac{1}{\frac{\lambda}{\sqrt{a}}\left(\frac{\alpha\sqrt{a}}{\lambda}+\sqrt{p}\right)\sqrt{p}}\right] e^{-\sqrt{p/a}x} =$$

$$= \frac{m_0 r}{\left(\alpha-\frac{k\lambda}{\sqrt{a}}\right)} \frac{e^{-\sqrt{p/a}x}}{\sqrt{p}(\sqrt{p}+k)} - \frac{m_0 r}{\left(\alpha-\frac{k\lambda}{\sqrt{a}}\right)} \frac{e^{-\sqrt{p/a}x}}{\sqrt{p}\left(\sqrt{p}+\frac{\alpha\sqrt{a}}{\lambda}\right)}.$$

Using the table of images [3], we obtain

$$L^{-1}\left[\frac{m_0 r}{\left(\alpha-\frac{k\lambda}{\sqrt{a}}\right)\sqrt{p}\left(\sqrt{p}+k\right)} e^{-\sqrt{p/a}x}\right] = \frac{m_0 r}{\left(\alpha-\frac{k\lambda}{\sqrt{a}}\right)}\left[\exp\left(k\frac{x}{\sqrt{a}}+k^2\tau\right)\operatorname{erfc}\left(k\sqrt{\tau}+\frac{x}{2\sqrt{a\tau}}\right)\right], \tag{21}$$

$$L^{-1}\left[\frac{m_0 r}{\left(\alpha-\frac{k\lambda}{\sqrt{a}}\right)\sqrt{p}\left(\sqrt{p}+\frac{\alpha\sqrt{a}}{\lambda}\right)} e^{-\sqrt{p/a}x}\right] = \frac{m_0 r}{\left(\alpha-\frac{k\lambda}{\sqrt{a}}\right)}\left[\exp(Hx+H^2a\tau)\operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}}+H\sqrt{a\tau}\right)\right]. \tag{22}$$

Since the Laplace transform has the property of being linear, the common original of Eq. (19) has the form

$$t(x,\tau) = t_0 + (t_a - t_0)\left[\operatorname{erfc}\frac{x}{2\sqrt{a\tau}} - \exp(Hx+H^2a\tau)\times\operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}}-H\sqrt{a\tau}\right)\right] - \frac{m_0 r}{\left(\alpha-\frac{k\lambda}{\sqrt{a}}\right)}\times \tag{23}$$

$$\times\left[\exp\left(k\frac{x}{\sqrt{a}}+k^2\tau\right)\operatorname{erfc}\left(k\sqrt{\tau}+\frac{x}{2\sqrt{a\tau}}\right) - \exp(Hx+H^2a\tau)\operatorname{erfc}\left(\frac{x}{2\sqrt{a\tau}}+H\sqrt{a\tau}\right)\right].$$

The solution of the initial problem is written in criterial form as follows:

$$\Theta_x = \left[\operatorname{erfc}\frac{1}{2\sqrt{\mathrm{Fo}_x}} - \exp(\mathrm{Bi}_x+\mathrm{Bi}_x^2\mathrm{Fo}_x)\operatorname{erfc}\left(\frac{1}{2\sqrt{\mathrm{Fo}_x}}+\mathrm{Bi}_x\sqrt{\mathrm{Fo}_x}\right)\right] -$$

$$-\Theta_m\left[\exp\left(\frac{\mathrm{Bi}_m\sqrt{\mathrm{Fo}_m}}{\sqrt{\mathrm{Fo}_x}}+\mathrm{Bi}_m^2\,\mathrm{Fo}_m\right)\operatorname{erfc}\left(\mathrm{Bi}_m\sqrt{\mathrm{Fo}_m}+\frac{1}{2\sqrt{\mathrm{Fo}_x}}\right)\right.$$
$$\left.-\exp\left(\mathrm{Bi}_x+\mathrm{Bi}_x^2\,\mathrm{Fo}_x\right)\operatorname{erfc}\left(\frac{1}{2\sqrt{\mathrm{Fo}_x}}+\mathrm{Bi}_x\sqrt{\mathrm{Fo}_x}\right)\right]. \qquad (24)$$

The obtained solution of (24) differs from the solutions provided by the literature on heat conductivity [3] by the existence of a second term in the right-hand part. This term characterizes the influence of the moisture exchange on the process of development and formation of a temperature field in the wall and in the adjacent ground massif.

The functions exp Z and erfc L are tabulated, and in the solution of actual problems of heating of walls, they can be determined with the aid of mathematical handbooks, e.g., [4].

NOTATION

t, temperature of the body; u, specific moisture content of the concrete; τ, time; a, thermal diffusivity of concrete; a_m, moisture diffusion coefficient of concrete; λ, heat conductivity; ζ, criterion of internal evaporation determining the amount of vapor diffused in the wall upon its being heated; r, latent heat of evaporation; c, specific heat capacity of concrete; δ, thermogradient coefficient characterizing moisture transfer due to the temperature gradient; t_0, mean initial temperature of the wall; t_a, temperature of the air supplied to the structure for drying the wall; α, heat-transfer coefficient; β, moisture-transfer coefficient based on the difference of the specific moisture contents; ρ, density of dry concrete; j, density of the heat flow from the evaporation surface; b, thickness of the concrete wall; u_e, equilibrium moisture content of concrete; u_0, initial specific moisture content of the concrete in the wall; $H=\alpha/\lambda$; $k=\beta/\sqrt{a_m}$; $\mathrm{Bi}_m=\beta b/a_m$; $\mathrm{Fo}_m=a_m\tau/b^2$; $\mathrm{Bi}_x=\frac{\alpha}{\lambda}x$; $\mathrm{Fo}_x=a\tau/x^2$; $\Theta_x=\frac{t(x,\tau)-t_0}{t_a-t_0}$; $\Theta_m=m_0r\Big/\left(\alpha-\frac{k\lambda}{\sqrt{a}}\right)(t_a-t_0)$.

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